

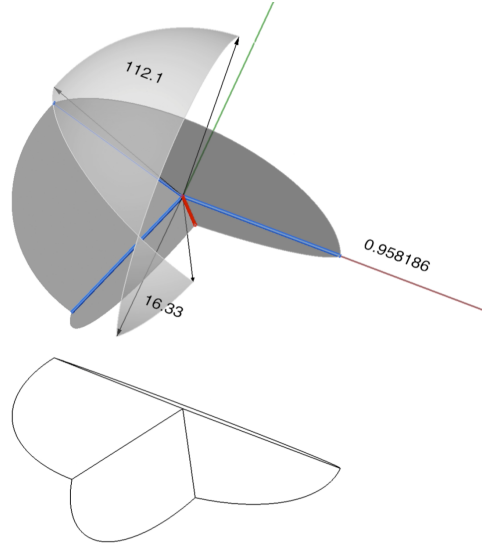
# Non-developable rigid foldable Quad Topology

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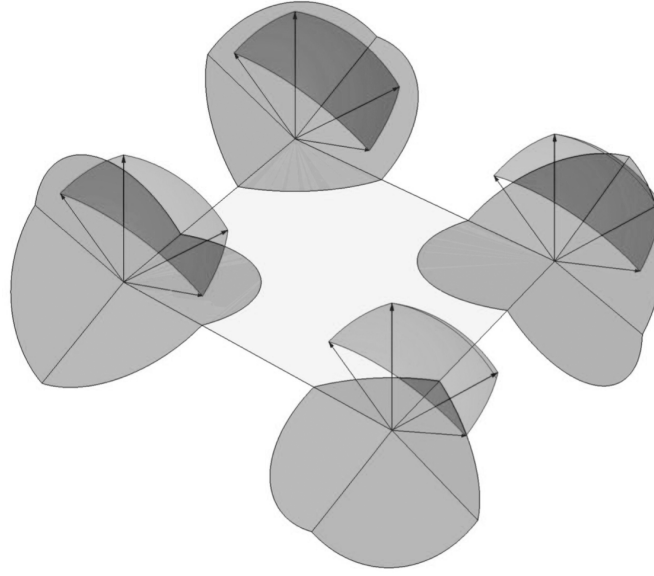
## Abstract

Technical origami offers great potential for the engineering of innovative applications in fields such as lightweight construction, adaptive structures and manufacturing processes. Although there is a solid base of knowledge surrounding the kinematics of rigid foldable origami, a generic process for generating specific shapes that are rigid foldable is yet to be created. These design targets require the generic understanding of the reciprocal transformability of rigid foldable piecewise geometries [Gattas and You 13].



**Figure 1:** Non-developable 4-fold vertex in an intermediate configuration with the corresponding trace on the gaussian sphere.

T.Tachi's 'Generalization of rigid-foldable quadrilateral-mesh origami' [Tachi 09] was an important step towards freedom in shaping foldable forms. Structurally feasible folding mechanisms however feature additional characteristics. They require a constrained motion, remain in a spatial build at all times and lack bifurcation as well as dead-center configurations [Buffart et al. 17]. Adding non-developable vertices to the parameter spectrum is technically advantageous, but renders a complexity to the development of a generic formfinding for rigid foldable mechanism arrays.



**Figure 2:** 4 Non-developable, interconnected and thus reciprocally foldable vertices, a 'loop'. The mechanism is shown with the corresponding traces on the gaussian sphere mapped onto each vertex

The theory is not new. A. Kokotsakis instigated to discuss the flexibility of a 'Neunflach', which can be translated literally as 'nine-flat', the 1DOF constrained mechanism consisting of 4 quadrilateral, planar, rigid facets joined flexibly in a 3x3 manner, in 1933 [Kokotsakis 32]. This is the elementary mechanism that folds only when underlying conditions are met. Hellmuth Stachel approached the loop character of this reciprocal condition kinematically using spherical mechanisms [Stachel 10]. Ivan Izvestiev depicts the comprehensive set of 8 solutions in his work 'Classification of flexible Kokotsakis polyhedra with quadrangular base' in [Izvestiev 16].

If we look at the Gaussian Curvature of non-developable vertices as Huffman does in 'Curvature and Creases' [Huffman 76], we naturally get a non-zero curvature. This is graphically represented as a single vertex in Fig 1 and as a visual instance of the smallest folding mechanism with a loop ('Neunflach') in Fig. 2. The remaining area Delta ' $\Delta$ ' of the enclosed spherical surface on the Gaussian sphere is independent from the facet angles. It is constant, furthermore it correlates with the deviation of the facet angle total to  $2\pi$ . The paper uses this unique feature of non-developable vertices to illustrate a different approach on solving loops in rigid foldable Quad Topologies using ' $\Delta$ '.

## References

- [Buffart et al. 17] H. Buffart, S. Hoffmann, J. paris, J. Siebrecht, Burkhard Corves, and M. Trautz. "Non-flat folding mechanisms for structural purposes." In *Proceedings of the IASS Annual Symposium 2017 - Interfaces: Architecture, Engineering, Science. September 25 - 28th, Hamburg, Germany*, edited by A. Bögle and M. Grohmann, 2017.
- [Gattas and You 13] J. Gattas and Z. You. "Comput. Aided Geom. Des. 2010." In *New Proposals for Trans-*

*formable Architecture, Engineering, and Design. Proceedings of the First Conference Transformables 2013. School of Architecture Seville, Spain, 18-20 September 2013*, edited by F. Escrig and J. Sanchez, pp. 319–324, 2013.

- [Huffman 76] D. Huffman. “Curvature and creases – A Primer on Paper.” *IEEE Transactions on Computers* C-25:10 (1976), 1010–1019.
- [Izmestiev 16] I. Izmestiev. “Classification of flexible Kokotsakis polyhedra with quadrangular base.” *International Mathematics Research Notices* 2017 :3 (2016), 715–808.
- [Kokotsakis 32] A. Kokotsakis. *Über bewegliche Polyeder*, 1932.
- [Stachel 10] H. Stachel. “A kinematic approach to Kokotsakis meshes.” *Comput. Aided Geom. Des.* 2010 :27 (2010), 428–437.
- [Tachi 09] T. Tachi. “Generalization of rigid-foldable quadrilateral-mesh origami.” *Journal of the International Association for Shell and Spatial Structures* 50.3 50:3 (2009), 173–179.